(Ref. 1, p. 24). We cannot share this view because of a) the aforementioned uncertainty of deductions based on cemented-burner data, b) our inability to discern an obvious maximum using uncemented burners, and c) the afterburning of fuel gas under conditions that should be lean according to this hypothesized stoichiometry. The fact that Burger failed to observe afterburning with methane (under conditions similar to those for which we observed it) is still, however, troublesome. Current investigations at our laboratory are aimed at resolving such questions of stoichiometry.

**References**

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## Comment on "Large-Amplitude Transverse Instability in Rocket Motors"

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THE coupling of longitudinal and transverse modes of high-frequency combustion instability was treated theoretically by Temkin[1,2] in an effort to explain the experimental observations of Crump and Price.[3] The object of this comment is to point out that mode coupling is not unique to solid-propellant rocket combustors, but also has been observed in gas-burning rockets.

Osborn and Bonnell[4,5] varied the cylindrical geometry of premixed gas rocket combustion chambers to determine the effect on high-frequency combustion pressure oscillations. It was found that when the length/diameter ratio was such that the frequencies of the fundamental longitudinal and tangential modes were approximately the same, the amplitudes of oscillation increased significantly. There was an accompanying "shift" of the instability region (on a plane of equivalence ratio vs combustion pressure) to a lower combustion pressure. The changes in the wave shapes of the pressure oscillations observed on an oscilloscope confirmed the belief that an interplay between the two aforementioned modes was occurring. The net effect of mode coupling in a premixed gas rocket was an increase in the severity of combustion instability.

**References**

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## Systems of First-Order, Nonlinear Differential Equations Convertible to Classical Forms

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IN a recent note, Mason[1] presented classes of first-order systems reducible to Bernoulli and Ricatti type of equations. The material in his note is a particular case of the work reported earlier in Ref. 2, a relevant extract of which is being presented here.

A general first-order nonlinear system can be described by the differential equation

$$\dot{x} + g(x,t) = 0 \qquad (1)$$

(dot denotes differentiation with respect to $t$).

The most general transformation law involving both the dependent and independent variables (unlike the transformation considered by Mason[1] involving the dependent variable only) can be written as

$$X = X(x,t) \qquad (2)$$

$$T = T(x,t) \qquad (3)$$

Differentiation of Eqs. (2) and (3) with respect to $t$ gives

$$X' = (X_x\dot{x} + X_t)/(T_x\dot{x} + T_t) \qquad (4)$$

where the subscripts $x$ and $t$ denote the corresponding partial derivatives and prime denotes differentiation with respect to $T$.

Substituting Eq. (1) in Eq. (4),

$$X' = (X_xg - X_t)/(T_xg - T_t) \qquad (5)$$

For Eq. (5), which represents the given system in the transformed plane, to be amenable to existing methods of analysis, it should conform to one of the classical forms, such as

Bernoulli type:

$$X' = C_1(T)X^n - C_2(T)X \ (n \neq 1) \qquad (6a)$$

($n = 0$ represents the linear type)

Ricatti type:

$$X' = C_1(T)X^2 + C_2(T)X + C_3(T) \qquad (6b)$$

Separable type:

$$X' = C_1(T)F(X) \qquad (6c)$$

Exact equation type:

$$X' = M(X,T)/N(X,T) \qquad (6d)$$

$$X' = M(X,T)/N(X,T) \qquad (6e)$$

(Here $\partial N/\partial T = \partial M/\partial X$.) Substitution of Eq. (5) into any one of Eqs. (6a–6d) results in a corresponding partial-differential equation. The solutions to such partial-differential equations represent the necessary transformation functions for a given first-order system, Eq. (1), to be reducible to one of the classical forms listed previously.

Although a unique general solution to these partial-differential equations in ($X$, $T$) may not be obtainable for any given function $g(x,t)$, it is useful even if a particular solution to any one of these partial-differential equations is obtained.

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